The method: Maximum and minimum

Set $f'(x) = 0$ to find the critical points. f'' negative: a maximum. f'' positive: a minimum.

$f(x) = -3x^3 + 9x^2 - 3 \quad \rightarrow \quad$ max at $x = 2$, min at $x = 0$

$f'(x) = -9x^2 + 18x = -9x(x - 2)$ set $f'(x) = 0$

critical points $x = 0$ and $x = 2$

$f''(x) = -18x + 18$

$f''(0) = 18$: min; $f''(2) = -18$: max f'' negative: a max; positive: a min

$f(x) = x^3 - 3x + 1 \quad \rightarrow \quad$ max at $x = -1$, min at $x = 1$

$f'(x) = 3x^2 - 3 = 3(x + 1)(x - 1)$ set $f'(x) = 0$

critical points $x = -1$ and $x = 1$

$f''(x) = 6x$

$f''(-1) = -6$: max; $f''(1) = 6$: min f'' negative: a max; positive: a min

Problems 1 and 2 worked out · Version 0A

$f(x) = x^3 - 9x^2 + 15x + 8 \quad \rightarrow \quad$ max at $x = 1$, min at $x = 5$

$f'(x) = 3x^2 - 18x + 15 = 3(x - 1)(x - 5)$ set $f'(x) = 0$

critical points $x = 1$ and $x = 5$

$f''(x) = 6x - 18$

$f''(1) = -12$: max; $f''(5) = 12$: min f'' negative: a max; positive: a min

$f(x) = 3x^3 + 27x^2 + 72x + 8 \quad \rightarrow \quad$ max at $x = -4$, min at $x = -2$

$f'(x) = 9x^2 + 54x + 72 = 9(x + 4)(x + 2)$ set $f'(x) = 0$

critical points $x = -4$ and $x = -2$

$f''(x) = 18x + 54$

$f''(-4) = -18$: max; $f''(-2) = 18$: min f'' negative: a max; positive: a min

Maximum and minimum · Version 0A

1) max at $x = 1$, min at $x = 5$
2) max at $x = -4$, min at $x = -2$
3) max at $x = 1$, min at $x = -3$
4) max at $x = -5$, min at $x = 5$
5) max at $x = 1$, min at $x = 5$
6) max at $x = 1$, min at $x = 3$
7) max at $x = 3$, min at $x = 5$
8) max at $x = 5$, min at $x = -5$
9) max at $x = -5$, min at $x = -3$

Answer key

10) max at x = 2, min at x = 4