The method: Inverse functions

Write y for f(x), swap x and y, and solve for y. The answer is $f^{-1}(x)$.

$f(x) = x^3 + 9 \rightarrow f^{-1}(x) = \sqrt[3]{x - 9}$

$y = x^3 + 9$ write y for f(x)

$x = y^3 + 9$ swap x and y

$x - 9 = y^3$ subtract 9

$f^{-1}(x) = \sqrt[3]{x - 9}$ solve for y

$f(x) = \frac{x + 2}{x - 3} \rightarrow f^{-1}(x) = \frac{3x + 2}{x - 1}$

$y = \frac{x + 2}{x - 3}$ write y for f(x)

$x = \frac{y + 2}{y - 3}$ swap x and y

$xy - 3x = y + 2$ multiply by the denominator

$xy - y = 3x + 2$ get the y terms on one side

$y(x - 1) = 3x + 2$ take out y

$f^{-1}(x) = \frac{3x + 2}{x - 1}$ solve for y

$f(x) = \frac{5x + 9}{4} \rightarrow f^{-1}(x) = \frac{4x - 9}{5}$

$y = \frac{5x + 9}{4}$ write y for f(x)

$x = \frac{5y + 9}{4}$ swap x and y

$4x = 5y + 9$ multiply by 4

$4x - 9 = 5y$ subtract 9

$f^{-1}(x) = \frac{4x - 9}{5}$ solve for y

Problems 1 and 2 worked out · Version 0A

$f(x) = \frac{4x + 6}{3} \rightarrow f^{-1}(x) = \frac{3x - 6}{4}$

$y = \frac{4x + 6}{3}$ write y for f(x)

$x = \frac{4y + 6}{3}$ swap x and y

$3x = 4y + 6$ multiply by 3

$3x - 6 = 4y$ subtract 6

$f^{-1}(x) = \frac{3x - 6}{4}$ solve for y

$f(x) = \frac{5x + 1}{3} \rightarrow f^{-1}(x) = \frac{3x - 1}{5}$

$y = \frac{5x + 1}{3}$ write y for f(x)

$x = \frac{5y + 1}{3}$ swap x and y

$3x = 5y + 1$ multiply by 3

$3x - 1 = 5y$ subtract 1

$f^{-1}(x) = \frac{3x - 1}{5}$ solve for y

Inverse functions · Version 0A

1) $f^{-1}(x) = \frac{3x - 6}{4}$

2) $f^{-1}(x) = \frac{3x - 1}{5}$

3) $f^{-1}(x) = \frac{5x - 7}{x - 1}$

4) $f^{-1}(x) = \frac{4x + 1}{5}$

5) $f^{-1}(x) = \frac{-x - 6}{x - 1}$

6) $f^{-1}(x) = \frac{2x - 9}{3}$

7) $f^{-1}(x) = \frac{6x - 7}{5}$