**The method: Rational roots**

Possible rational roots are ± a factor of the constant over a factor of the leading coefficient. Find one that works, divide, and factor.

$x^3 + 4x^2 - 9x - 36 = 0$ $x = -4$ or $x = -3$ or $x = 3$

try ±1, ±2, ±3, ±4, ±6, ±9, ±12, ±18, ±36

factors of the constant over factors of the leading coefficient

| $-4$ | $1$ | $4$ | $-9$ | $-36$ |
|---|---|---|---|---|
| | | $-4$ | $0$ | $36$ |
| | $1$ | $0$ | $-9$ | $0$ |

$x = -4$ gives remainder 0

$(x + 4)(x^2 - 9) = 0$ the bottom row is the other factor

$(x + 4)(x - 3)(x + 3) = 0$ factor the quadratic

$x = -4$ or $x = -3$ or $x = 3$

$2x^3 + 5x^2 - 9x - 18 = 0$ $x = -3$ or $x = -\frac{3}{2}$ or $x = 2$

try ±1/2, ±1, ±3/2, ±2, ±3, ±9/2, ±6, ±9, ±18

factors of the constant over factors of the leading coefficient

| $-3$ | $2$ | $5$ | $-9$ | $-18$ |
|---|---|---|---|---|
| | | $-6$ | $3$ | $18$ |
| | $2$ | $-1$ | $-6$ | $0$ |

$x = -3$ gives remainder 0

$(x + 3)(2x^2 - x - 6) = 0$ the bottom row is the other factor

$(x + 3)(2x + 3)(x - 2) = 0$ factor the quadratic

$x = -3$ or $x = -\frac{3}{2}$ or $x = 2$

$3x^3 - 4x^2 - 49x - 30 = 0$ $x = -3$ or $x = -\frac{2}{3}$ or $x = 5$

try ±1/3, ±2/3, ±1, ±5/3, ±2, ±3, ±10/3, ±5, ±6, ±10, ±15, ±30

factors of the constant over factors of the leading coefficient

| $5$ | $3$ | $-4$ | $-49$ | $-30$ |
|---|---|---|---|---|
| | | $15$ | $55$ | $30$ |
| | $3$ | $11$ | $6$ | $0$ |

$x = 5$ gives remainder 0

$(x - 5)(3x^2 + 11x + 6) = 0$ the bottom row is the other factor

$(x - 5)(3x + 2)(x + 3) = 0$ factor the quadratic

$x = -3$ or $x = -\frac{2}{3}$ or $x = 5$

# Answer key

**Problems 1 and 2 worked out · Version 0A**

$2x^3 + 13x^2 + 13x - 10 = 0$ $x = -5$ or $x = -2$ or $x = \frac{1}{2}$

try ±1/2, ±1, ±2, ±5/2, ±5, ±10 factors of the constant over factors of the leading coefficient

| | | | | |
|---|---|---|---|---|
| −5 | 2 | 13 | 13 | −10 |
| | | −10 | −15 | 10 |
| | 2 | 3 | −2 | 0 |

$x = -5$ gives remainder 0

$(x + 5)(2x^2 + 3x - 2) = 0$ the bottom row is the other factor

$(x + 5)(2x - 1)(x + 2) = 0$ factor the quadratic

$x = -5$ or $x = -2$ or $x = \frac{1}{2}$

$x^3 - 19x + 30 = 0$ $x = -5$ or $x = 2$ or $x = 3$

try ±1, ±2, ±3, ±5, ±6, ±10, ±15, ±30

factors of the constant over factors of the leading coefficient

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 0 | −19 | 30 |
| | | 3 | 9 | −30 |
| | 1 | 3 | −10 | 0 |

$x = 3$ gives remainder 0

$(x - 3)(x^2 + 3x - 10) = 0$ the bottom row is the other factor

$(x - 3)(x - 2)(x + 5) = 0$ factor the quadratic

$x = -5$ or $x = 2$ or $x = 3$

**Rational roots · Version 0A**

1) $x = -5$ or $x = -2$ or $x = \frac{1}{2}$
2) $x = -5$ or $x = 2$ or $x = 3$
3) $x = -2$ or $x = \frac{1}{3}$ or $x = 2$
4) $x = -\frac{1}{3}$ or $x = 4$ or $x = 5$
5) $x = -5$ or $x = -\frac{1}{3}$ or $x = 5$
6) $x = -2$ or $x = \frac{1}{3}$ or $x = 3$
7) $x = -4$ or $x = -\frac{1}{2}$ or $x = 3$
8) $x = -4$ or $x = -3$ or $x = -\frac{3}{2}$